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A HIGH-RESOLUTION
SPECTRAL ANALYSIS TECHNIQUE



D. R. E. A. TECHNICAL MEMORANDUM 83/D

RESEARCH AND DEVELOPMENT BRANCH
DEPARTMENT OF NATIONAL DEFENCE
CANADA

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A HIGH-RESOLUTION
SPECTRAL ANALYSIS TECHNIQUE

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ABSTRACT

A modified version of the complex demodulation approach is used in the development of an efficient high-resolution technique for spectral analysis. Through a combined filtering and decimation operation and a suboptimal Finite Impulse Response (FIR) digital filter, the Fast Fourier Transform (FFT) sizes in the fast convolutions are reduced significantly. Though this procedure introduces wraparound effects, such errors are small compared to aliasing arising from the normal digital filtering and decimation operations. Aside from its capability to analyze multiple finite bandwidths with different frequency resolutions, the accuracy obtainable by this method allows for a large analysis dynamic range.

RESUMÉ

Le présent article porte sur une version modifiée de l'approche complexe de la démodulation appliquée au développement d'une technique efficace d'analyse spectrale à haute résolution. La combinaison du filtrage et de la décimation, et l'utilisation d'un filtre numérique sous-optimal à réponse en impulsions finies (RIF) permettent de réduire sensiblement la transformée de Fourier dans le calcul rapide des convolutions. Bien que cette technique introduise des erreurs globales, ces dernières sont petites par rapport à celles résultant du filtrage d'harmoniques (filtrage numérique et décimation). A part les analyses de plusieurs largeurs de bande finies avec différentes résolutions en fréquence, cette technique, par sa précision, permet des analyses dans une gamme dynamique étendue.

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1. INTRODUCTION

The facility to "zoom" in on selected narrow frequency bands with high resolution for a wide-band spectrum has many applications including sonar and vibration analyses. With the traditional FFT method, resolution is directly proportional to the length of the input sequence being processed. In practice, however, the length of the sequence that can be analyzed by available FFT processors is limited. As a result, the long input sequence (required for high resolution) is normally subdivided into smaller records (possibly overlapping) of equal length compatible with the capability of the available FFT processor. Such smaller records are then processed, normally by FFT and other standard digital processing operations, to provide the required high resolution equal to or close to that of the traditional FFT method. To achieve this zoom capability, a number of specialized techniques have been developed.

Shaw [1] and Gilchrist [2] proposed a zoom transform that was modified later by Yip [3]. In this technique, the input sequence length, say N , is a composite number Mm . FFTs of sequential sub-records of length M are first computed and followed by a smaller FFT of length m after some simplifying assumptions to reduce computations. Next, modifications are introduced to correct the estimates for a sinusoidal input. However, this transform does not calculate the narrow-band spectrum exactly, and introduces artifacts. Moreover, no means are given to predict when the zoom's accuracy is sufficient.

Nuttall [4] proposed a high-resolution FFT approximation referred to as vernier spectral analysis that uses strongly overlapping (50% and greater) sub-records in order to reduce aliasing errors through suitable windowing procedures first on the temporal segments and then on the delayed spectral results over the narrowband of interest. Fjell and Lunde [5] modified this technique to reduce the aliasing as well as the amount of overlap and referred to it as the cascade FFT. Still the overlap required for acceptable results is substantial (around 25%), so that the computational load could still be high.

Liu and Mintzer [6], in a direct approach, used direct decimation of the broad-band input sequence to reduce both the computation rate and storage requirements. They derived an expression for the expected signal-to-noise ratio and incorporated it in the design rules for the decimating finite-impulse-response (FIR) filter. Cooley and Winograd [7] relaxed the constraints on the pass-band of the decimating filter to reduce the amount of computation. This method is advantageous when direct operation on the raw input data is convenient and the narrow band of interest is preselectable.

To obtain error-free estimates for high resolution requirements, the chirp z transform (CZT) [8] is superior to the above-mentioned methods. Ferrie, Nawrocki, and Carter [9] utilized a partitioned and modified chirp z transform (PAM-CZT) to effect a savings in transform size and memory and allow computation of a large FFT [10-12] by means of several smaller FFTs. Even though this procedure can be adapted to examine narrow-bands with high resolution, the attendant computational load remains high when compared to other approximation methods.

Finally, a viable alternative to the above methods is the complex demodulation approach, the subject of this paper. Essentially, it is equivalent to the analog procedure of heterodyning the frequency band of interest to baseband, filtering and decimating the filter output, and then carrying out a spectral analysis of the decimated signal. The digital signal processing equivalent has been discussed by Bingham, Godfrey and Tukey [13] for the purpose of spectral analysis; outlined by Peled and Liu [14] in their book on digital signal processing; compared from the computational aspect by Hoyer and Stork [15] to Yip's method [3], and by Liu and Mintzer [6] with their direct narrow-band decimation technique. Walkty and Pednault [16] have examined the method in detail to preserve the equivalence with the analog procedure. Recently, Hung [17] has published an algorithm that is similar to the complex demodulation approach; though the algorithm is efficient in computation and storage, the analysis is confined to the central half of the band selected for higher resolution and allows for a dynamic range of about 30 dB.

This paper introduces simplifications in the complex demodulation technique with little loss in accuracy compared with the results of the traditional FFT approach and with little loss in analysis flexibility for practical applications. The simplifications introduced here are similar to those suggested by Pelkowitz [18-20], who focused his attention on a rigorous theoretical analysis based on periodic time-varying filters. This paper, however, shows that a suboptimal approach yields excellent practical results. A significant reduction in computation results from the combined filtering and decimation operation and parallel spectral analysis of multiple finite bandwidths is possible. With simple filter design procedures, the filter sidelobe level appears to be the main factor limiting the achievable analysis dynamic range; in fact, dynamic ranges exceeding 60 dB are readily attainable.

2. COMPLEX DEMODULATION TECHNIQUE

2.1 Data Segmentation

In practice, hardware/software and storage limitations impose constraints on the amount of data that can be processed at any one

time. For example, the FFT size might be limited to 4096 complex points which places a limit on the spectral resolution unless successive (possibly overlapping) data segments are suitably processed to give the required higher resolution. As a result, consider a sampled input sequence $\{x(n)\}$ with sampling rate equal to f_s . The segmentation of $\{x(n)\}$ depends on the amount of overlap needed for other signal processing requirements. For example, if frequency-domain beamforming via the FFT technique is used for a linear array, then it might be desirable to form beams with about 5-12% overlap to provide reasonable search in specified directions. Therefore, to allow for such requirements, consider overlapping data segments $x_l(n)$ defined by

$$x_l(n) = x(n+l(N-M+1)) \quad (1)$$

where

$l = 0, 1, 2, \dots$ and denotes the segment number;

$n = 0, 1, \dots, N-1$ denotes the sample number in each segment consisting of N samples;

$M-1$ denotes the amount of overlap from segment to segment and M takes on any integer value between 1 (no overlap) to N (overlap by $N-1$ points).

Figure 1 illustrates this segmentation scheme.

2.2 Complex Demodulation Principle

Complex demodulation can be considered to be a method of producing a signal that is the low-frequency image of one occupying a higher frequency band of interest. Computationally, the entire spectrum is shifted so that the center of the band of interest is moved down to zero frequency, the result run through a lowpass filter to isolate the band of interest, and finally the filter output is decimated to remove redundant samples.

Essentially, the complex demodulation operations are as follows:

a. Complex Modulation: If the frequency band of interest is centered about f_0 , then the spectrum is shifted down until f_0 coincides with 0. This result is obtained by multiplying the original input $x(n)$ by the complex sinusoid $\exp[-j2\pi f_0 n]$ to give the complex modulate

$$x_c(n) = x(n)\exp[-j2\pi f_0 n] \quad (2)$$

b. Lowpass Filtering: If $h(n)$ is the unit impulse response of the lowpass filter that passes all frequencies in the band of

interest, then the filtered output $x_F(n)$ is the convolution of $x_C(n)$ with $h(n)$: that is

$$x_F(n) = x_C(n) * h(n), \quad (3)$$

where the symbol $*$ denotes discrete convolution.

c. Decimation: The filtered output is decimated by an integer factor D to remove redundant samples and keep aliasing in the band of interest to a minimum: that is, every D th sample of $x_F(n)$ is retained to give the decimated output

$$x_D(n) = x_F(nD). \quad (4)$$

The DFT of any given length of $x_D(n)$ gives a resolution that is D times better than that of $x(n)$ of the same length. Steps a, b and c constitute a zoom transform over a finite bandwidth.

2.3 Economical Complex Demodulation

For practical purposes, the set of operations (complex modulation, lowpass filtering, and decimation) must be carried out as economically as possible and with an accuracy acceptable to the intended application to produce the same end result as in the preceding section.

a. Complex Modulation:

Complex modulation is readily carried out in the frequency domain thus. Let $x_{C\ell}(n)$ be the signal obtained when the segment $x_\ell(n)$ is complex modulated by $\exp[-j2\pi f_0 n]$: that is

$$x_{C\ell}(n) = x_\ell(n) \exp[-j2\pi f_0 (n + \ell(N-M+1))].$$

The DFT (Discrete Fourier Transform) of $x_{C\ell}(n)$ is given by

$$X_{C\ell}(k) = W_N^{\ell(k_0 + \delta)(N-M+1)} \sum_{n=0}^{N-1} x_\ell(n) W_N^{n(k+k_0 + \delta)}$$

$$\text{where } W_N = \exp[-j2\pi/N] \quad (5)$$

$$f_0 = (k_0 + \delta)f_s/N \quad (6)$$

$$k_0 = \text{positive integer, and } |\delta| < 1$$

and $f_s = \text{sampling frequency.}$

Without much loss in flexibility for practical applications, f_0 can be chosen to be an exact multiple of f_s/N since N is normally sufficiently large (2048 or 4096, for example). Therefore, if $X_\ell(k)$ denotes the DFT of x_ℓ , the use of the identity $W_N^{\ell k_0 N} \equiv 1$ and of $\delta=0$ gives the result

$$X_{Cl}(k) = W_N^{-lk_0(M-1)} X_l((k+k_0))_N \quad (7)$$

where $((k+k_0))_N$ denotes a circular shift to the left by k_0 units on the frequency index k .

Hence, by choosing f_0 to be an integer multiple of f_s/N , the complex modulation operation is obtainable by rotating the complex spectrum $X_l(k)$ to the left by k_0 bins and applying a phase factor.

b. Lowpass Filtering

Next the modulated segments $X_{Cl}(k)$ are processed by a lowpass filter to produce an unbroken (non-segmented) output sequence $x_F(n)$. In practice, the amount of overlap between the segments $x_l(n)$ can vary between 0 and 50%. For example, in sonar applications, the amount of overlap is around 5-12% of N to allow for phaseshift beamforming in the frequency domain. With such overlapping segments, the filtering operation can be carried out economically using the fast convolution "overlap-save" method [21].

Effective FIR (Finite Impulse Response) lowpass filters can be designed by a number of techniques, one of which is described briefly in Appendix A. Let $h(n)$ denote the unit impulse response of the FIR filter, where $n=0, 1, \dots, M_F-1$ and M_F is the filter length which is less than N . Then the overlap-save method corresponds to implementing a circular convolution of $h(n)$ with $x_{Cl}(n)$ and identifying the part that corresponds to a linear convolution. In particular, in this method of convolution of the M_F -point unit impulse response with an N -point segment, the first M_F-1 points are invalid whereas the remaining $N-M_F+1$ points are identical to those of a linear convolution. Therefore, for applications of practical interest involving segment overlaps around 5-12%, choosing $M_F=M$ corresponds to near optimum filter length for the filtering operation by fast convolution [22].

The filtering operation described above is carried out in the frequency domain as follows:

1. Let $H(k)$ be the N -point DFT of $h(n)$ with $N-M$ zeros added to the M values of $h(n)$. In practice, the $H(k)$'s would be precomputed and stored for the few filters required for higher resolution analysis.

2. Form the filter output

$$X_{Fl}(k) = X_{Cl}(k) H(k), \quad k=0,1,\dots,N-1 \quad (8)$$

for successive l values, say $l=0,1,\dots,L-1$. The substitution of (7) into (8) gives

$$X_{F\ell}(k) = W_N^{-\ell k_0(M-1)} X_{\ell}((k+k_0))_N H(k). \quad (9)$$

To reduce the computational load with hardly any loss in analysis flexibility, the constraint

$$k_0(M-1) = \text{multiple of } N \quad (10)$$

is imposed. With this choice of k_0 and M , the phase factor $W_N^{\ell k_0(M-1)}$ is equal to unity identically. For example, if $N=4096$ and $M=257$ then the choice of $k_0=\text{multiple of } 16$ gives the desired result. Thus, with this constraint, the phase factor does not enter into the computations at any stage, and (9) simplifies to

$$X_{F\ell}(k) = X_{\ell}((k+k_0))_N H(k) \quad (11)$$

3. Now take the N -point IDFT (Inverse Discrete Fourier Transform) of (11) to obtain

$$x_{F\ell}(n) \text{ where } n=0,1,\dots, N-1,$$

and save only the last $N-M+1$ valid points of each segment.

4. Concatenate the valid points in Step 3 to get the unbroken filter output

$$x_F(m) = x_F(n+\ell(N-M+1)-M+1) = x_{F\ell}(n) \quad (12)$$

where $n=M-1,\dots,N-1$

and $\ell=0,1,\dots,L-1$

The number of segments L to be processed depends on the decimation factor D and the resolution required, which are considered next.

c. Decimation

The filter frequency response must be designed to effectively isolate the narrow frequency band of interest and allow for decimation of the filter output $x_F(m)$ by the factor D , say, to produce the new sequence

$$x_D(m) = x_F(Dm). \quad (13)$$

The operation in (13) must remove redundant samples in $x_F(m)$ by keeping negligible the frequency aliasing arising from decimation.

The possible aliasing arises thus. If $X_F(e^{j\Omega T})$ denotes the Fourier transform of $x_F(m)$, where Ω is the radian frequency and T the sampling period of $x_F(m)$, then the Fourier transform of $x_D(m)$ is related to that of $x_F(m)$ [23] by:

$$X_D(e^{j\Omega T'}) = \frac{1}{D} \sum_{i=0}^{D-1} X_F(e^{j(\Omega T - 2\pi i/(D))}) , \quad (14)$$

where $T' = DT$ is the sampling period of $x_D(m)$. Furthermore, if $X(e^{j\Omega T})$ and $H(e^{j\Omega T})$ denote the Fourier transforms of $x(n)$ and $h(n)$ respectively, then they are related to $X_F(e^{j\Omega T})$ by

$$X_F(e^{j\Omega T}) = X(e^{j(\Omega - \Omega_0)T})H(e^{j\Omega T}) \quad (15)$$

where Ω_0 is the radian frequency corresponding to f_0 . As a result, the expected spectral structure of $x(n)$ and the required increase in frequency resolution directly affect the filter design and the decimation factor D .

Figure 2 illustrates schematically the possible aliasing arising from decimation. In this example, Figure 2a shows the spectrum of the input $x(n)$ after heterodyning to baseband; 2b shows the result after low pass filtering of $x(n)$; 2c shows negligible aliasing for a decimation factor of $D=3$; whereas 2d shows unacceptable aliasing for a decimation factor of $D=4$.

d. Frequency Resolution

In the preliminary FFT's of the individual segments $x_l(n)$, the frequency resolution without windowing in the time domain or smoothing in the frequency domain is equal to f_s/N . To improve the frequency resolution by the factor D by the complex demodulation method, the input in the band about f_0 (of interest) is heterodyned down to baseband, filtered by a lowpass filter extending from $-f_s/(2D)$ to $f_s/(2D)$, and decimated by a factor D to produce a sequence $x_D(m)$ of length N , which is finally analyzed by FFT means.

In building up the sequence $x_D(m)$ from the individual segments $x_l(n)$, the FFT of $x_D(m)$ - of length less than N - may be carried out at intermediate stages to interactively monitor the improvement with increasing resolution as L increases. The frequency resolution of $x_D(m)$ formed from L segments of $x_l(n)$ is $f_s/[L(N-M+1)]$. That is, the improvement in frequency resolution over that of a single segment is $L(N-M+1)/N$. To produce the sequence $x_D(m)$ of length N requires that $L_{\max}$ segments be processed, where

$$L_{\max} = \text{integer} \geq D(N-1)/(N-M+1). \quad (16)$$

The spectrum of $x_D(m)$ of length N has a resolution equal to $f_s/(ND)$.

e. Further Economy: Combined Filtering and Decimation

A substantial saving in the computations in Steps b and c above can be effected by combining the lowpass filtering and decimation in one operation. By means of the Sampling Theorem (Theorem 9) of Cooley, Lewis and Welch [24],

$$x_{F\ell}(nD) = \text{IDFT of } \sum_{k=0}^{D-1} x_{F\ell}(k + \frac{iN}{D}) \quad (17)$$

for $n=0,1,\dots, N/D - 1$.

The overlapped and summed version of $x_{F\ell}(k)$ on the right side of (17) is an aliased version of $x_{F\ell}(k)$; and the Inverse Discrete Fourier Transform of this aliased spectrum gives exactly the values of $x_{F\ell}(n)$ at every Dth point. From equation (11), namely

$$x_{F\ell}(k) = x_{\ell}((k+k_0))_N H(k),$$

the frequency response $H(k)$ of the lowpass filter can be designed so that the values of $H(k)$ and of $x_{F\ell}(k)$ in the stopband

$$k = N/(2D), N/(2D) + 1, \dots, N - N/(2D) - 1$$

are extremely small relative to those in the passband

$$k = 0,1,\dots, N/(2D) - 1 \text{ and } k = N - N/(2D), \dots, N-1.$$

As a result, a simple yet effective approximation to the right side of (17) is

$$x_{F\ell}(nD) \approx N/D\text{-point IDFT of } x_{F\ell}^{\dagger}(k) \quad (18)$$

where

$$x_{F\ell}^{\dagger}(k) = \begin{cases} x_{F\ell}(k) & \text{for } k=0,1,\dots, N/(2D) - 1 \\ x_{F\ell}(N-N/D + k) & \text{for } k = N/(2D), N/(2D) + 1, \dots, N/(D)-1 \end{cases} \quad (19)$$

The use of (18) and (19) allow for combined filtering and decimation with a transform size of length N/D , instead of filtering using a transform of length N followed by subsampling.

The additional constraints required in (18) and (19) are simply:

(i) N must be divisible by $2D$ in order to form $x_{F\ell}^{\dagger}(k)$ in (19); and

(ii) $M-1$ must be divisible by D because, in forming the unbroken decimated filtered output from (18), the first $(M-1)/D$ points of $x_{F\ell}(nD)$ are invalid and the last $(N-M+1)/D$ points of $x_{F\ell}(nD)$ for $\ell=0,1,\dots,L-1$ are concatenated to give $x_D(m)$.

If a radix-2 FFT algorithm is used to compute the DFT's, then approximately $N(\log_2 N + 1)$ complex operations are required to form the product in (11) and compute $x_{F\ell}(n)$ from $x_{F\ell}(k)$. In comparison, only $(N/D)(\log_2 N - \log_2 D + 1)$ complex operations are required to

compute $x_{F\ell}(nD)$ from $x_{F\ell}^+(k)$. Hence, the use of (18) reduces the computational load for filtering and decimation by the factor

$$F = D(\log_2 N + 1) / (\log_2 N - \log_2 D + 1) \quad (20)$$

Values of F for $D=2, 4, 8$ and 16 and $N=1024, 2048$ and 4096 are given below in Table I. As an example, for $D=8$, the computational load for filtering and decimation is reduced by at least one order.

TABLE I: REDUCTION IN COMPUTATIONAL LOAD
FOR FILTERING AND DECIMATION

N \ D	2	4	8	16
1024	2.4	5.5	12.6	29.3
2048	2.4	5.3	12.0	27.4
4096	2.4	5.2	11.6	26.0

3. COMPUTATIONAL LOAD

Computational efficiency is an important consideration in real-time applications. In practice, various segments of the input spectrum from different data channels undergo spectral analysis with different frequency resolutions. For example, in sonar applications, as many as eight bands of variable widths might be selected from the input spectrum for higher resolution analysis.

A simple expression for the number of computations required by the proposed frequency-domain approach to process K bands using a decimation factor D to achieve a frequency resolution of $f_s/(ND)$ is:

$$LA + LKB + AK \quad (21)$$

where

$$(i) \quad L = D(N-1)(N-M+1)^{-1}$$

denotes the number of input overlapping data segments that must be processed to obtain the desired frequency resolution;

$$(ii) \quad A = N \log_2 N,$$

in the first term, denotes the number of complex operations required to transform each input data segment to the frequency domain by the familiar FFT algorithm;

$$(iii) \quad B = (N/D) + (N/D) \log_2 (N/D)$$

denotes the number of complex operations required for the combined filtering and decimation of each data segment by means of the fast convolution ("overlap and save") method;

(iv) and A in the last term accounts for the number of complex operations required by the final FFT on the concatenated (filtered and decimation) sequence to produce the high resolution spectrum for each band of interest. Appropriate sums of the second and third terms are involved in the analysis of multiple bands with varying resolutions.

To illustrate computational efficiency, the number of operations required with a floating-point unit using pipelined arithmetic hardware (typical of real-time signal processors available today) will be presented and compared with the results of the long (i.e. conventional) FFT approach. In such hardware, one operation refers to one complex multiply, or one complex add, or one complex multiply-add. For the FFT on sequences of length N, the standard $N \log_2 N$ expression is used to determine the number of operations. Table II shows the number of operations in units of 10^3 for the long FFT method and the frequency-domain approach in this paper for a representative number of bands and resolution widths, as indicated in the two central columns. For an input sampling rate of 4096 samples/sec on each channel and FFT segments of length 4096 overlapping by about 6%, the first and last columns indicate that the frequency domain approach is at least comparable to the efficient long FFT method. Essentially, higher resolution has been exchanged for smaller analysis bandwidths. The high resolution technique, however, requires storage ranging from 10 to 50% of that for the long FFT method and can be implemented on hardware processors having limited FFT lengths.

TABLE II
OPERATIONS FOR SPECTRAL ESTIMATION (10^3)

COMPLEX DATA:

<u>LONG FFT METHOD</u>	<u>NO. OF BANDS</u>	<u>RESOLUTION WIDTHS (HZ)</u>	<u>FREQ-DOMAIN METHOD</u>
492	1	1/8	512
827	1	1/2, 1/4, 1/8	711
1876	1	1/2, 1/4, 1/8, 1/16	1216
1876	2	1/2, 1/4, 1/8, 1/16	1592

REAL DATA:

938	1	1/2, 1/4, 1/8, 1/16	799
938	2	1/2, 1/4, 1/8, 1/16	1179

4. ANALYSIS DYNAMIC RANGE

Aside from computational efficiency, analysis dynamic range (i.e. numerical accuracy) is another important consideration in practical applications. For example, in sonar applications, a spectral dynamic range of about 60 dB is required in cases where the desired signal is contaminated by strong nearby interferences. As a result, a brief description of the sources of error in the spectral estimation procedure will identify the factors limiting the analysis dynamic range.

The sources of error in the complex demodulation approach lie in the filtering and decimation operations. In the conventional digital filtering and decimation procedures, involving a linear time-invariant filter whose impulse response length does not exceed the number of overlapping data points by more than one, the sources of error are as follows.

One source of error in the calculated spectrum is due to the passband ripples of the lowpass filter $H(e^{j\omega T})$. Technically, the

spectral output should be multiplied by an appropriate correction factor to ensure unity gain in the filter passband. For the Kaiser type FIR filter mentioned in Appendix A, the passband ripple is around ± 0.0009 dB for the design parameter β of 7.865 that gives a maximum sidelobe level of -80 dB. In the case of such small passband ripples, this correction factor may be ignored.

The second and main source of error is the aliasing caused by the decrease in the sampling rate by the factor D . This accounts for the purpose of the lowpass filter $H(e^{j\omega T})$, namely to suppress as much as possible the aliasing arising from decimation. Arguments similar to those of Liu and Mintzer [6] can be used to derive a signal-to-aliasing noise power ratio defined by

$$\text{SNR} = \frac{|\text{signal}|^2}{|\text{aliasing error}|^2} \quad (22)$$

Of interest, however, is the following lower bound: namely,

$$\text{SNR} \geq \left[\frac{1 - \delta_p}{(D-1)\delta_s} \right]^2, \quad (23)$$

where δ_p and δ_s are respectively the maximum passband ripple and maximum stopband ripple of the filter $H(k)$. For example, with $1 - \delta_p = 0.0009$ dB, $\delta_s = -80$ dB and $D=8$, the value of $\text{SNR} \geq 63$ dB: that is, in this case, the analysis dynamic range can be expected to be at least 60 dB.

In the combined filtering and decimation approach, however, an additional source of error comes into play. In this approach, the linear time-invariant filter $H(k)$ is replaced by a sampled-data FIR filter whose frequency response is

$$H^\dagger(k) = \begin{cases} 0 & \text{in the stopband at } k = N/(2D), \dots, N - N/(2D) - 1 \\ H(k) & \text{in the passband at } k=0, \dots, N/(2D) - 1 \text{ \& } N - N/(2D), \dots, N-1. \end{cases}$$

This means that the sampled-data FIR filter $H^\dagger(k)$ has an impulse response of length N , compared to that of $H(k)$ with an impulse response of length M . Figures 3 and 4 illustrate the differences between the two filters in their impulse responses (with magnitude plotted as a continuous curve on a dB scale) and in their frequency responses, for a Kaiser filter $H(k)$ of length $M=257$ and maximum sidelobe level of -80 dB. The impulse response $h^\dagger(n)$ of the sampled-data filter, in Fig. 4, is of length N and is nearly identical with that in Fig. 3 but for an additional component which is extremely small and is about 95 dB or more below the maximum value of $h(n)$. In effect, this small component resembles a "tail" attached to the main response in $h(n)$ and gives rise to a circular convolution error in the filtering operation. Also, the frequency response $H^\dagger(k)$ in Fig. 4 still resembles that of $H(k)$ in Fig. 3, except that the sidelobes of $H^\dagger(k)$ have a finer structure.

In addition, the envelopes to the sidelobes of $H^+(k)$ and $H(k)$ are practically identical. As a result, the combined filtering and decimation operation still gives rise to an aliasing error that yields practically the same lower bound as in (23).

More rigorously, Pelkowitz [18-20] has shown that "wraparound" (or circular convolution) error occurs when the length of the overlap is less than the filter impulse length minus one. Moreover, this error transforms a linear time-invariant filter into a periodic time-varying filter, whose output is periodically nonstationary for a wide-sense stationary input. Furthermore, the "tail" in the impulse response of $h^+(n)$ gives rise to additional aliasing in the filtering and decimation operation. However, for the suboptimal filter suggested in this paper (i.e. the sampled-data filter having a Kaiser frequency response in the passband), the results to be given below indicate that this additional aliasing arising from the small "tail" is a second order effect compared to the conventional aliasing arising from the familiar filtering and decimation operations.

Figure 5 illustrates the higher resolution analysis obtained with the filter $H^+(k)$ in Fig. 4 with $D=8$, for the case of 5 sinusoids located between frequency cells and spanning a dynamic range of 70 dB and for floating point operations having 24 bit mantissa. The top figure shows the 5 sinusoids all correctly resolved and having the correct relative amplitudes for the following input:

<u>Normalized Frequency</u>	<u>Amplitude Level</u>	<u>Amplitude in dB</u>
.248 3215	.001	-60 dB
.253 2043	1.0	0 dB
.258 0872	.0003162	-70 dB
.277 6184	1.0	0 dB
.282 5012	.001	-60 dB

The bottom figure shows the absolute error in dB relative to the direct (long) FFT result. In this analysis, smoothing by Hanning weights was employed in the frequency domain. Thus, for floating point operations with 16 bit mantissa and for Hanning smoothing, the analysis dynamic range should be around 80 dB - and this result was confirmed through simulations using the TASP hardware processor [25] at DREA.

Figure 6 shows the unsmoothed results corresponding to the same example as in Fig. 5 - that is, using the filter $H^+(k)$ in Fig. 4 on the five sinusoids mentioned above. These results, representing a typical worse case situation, illustrate three interesting facts: (a) the result in Fig. 6(a) shows the two main peaks having unity amplitude while the other peaks become evident after smoothing is employed, as illustrated in Fig. 5; (b) the maximum absolute error in the unsmoothed result in Fig. 6(b), relative to the direct FFT computation, is -62 dB which agrees well with the simple lower bound in (23); (c) the absolute error in the unsmoothed result in Fig. 6(c), relative to the

conventional filtering and decimation operation is less than -95 dB and confirms the point that the aliasing arising from the "tail" of the impulse response $h^\dagger(n)$ is indeed a second order effect compared to the aliasing arising from the conventional filtering and decimation operation. In fact, the error in Fig. 6(b) is due almost totally to this type of aliasing and found to be so from actual simulations.

5. SUMMARY OF IMPLEMENTATION

The basic steps in implementing this high resolution spectral analysis technique are as follows:

1. Subdivide the input data into overlapping segments as in Fig. 1. Each segment is of length N and overlaps each adjacent segment by $M-1$ points: that is, the segments are

$$x_l(n) = x(n + l(N-M+1))$$

where $n=0,1,\dots,N-1$ and $l=0,1,\dots,L-1$

Example: $N=2048$ and $M-1=256 \approx 12\%$ of N .

2. Form the N -point FFT of each segment; that is, form

$$X_l(k) = N\text{-point FFT of } x_l(n) .$$

3. Select the center frequency index k_0 of the frequency band of interest so that

$$k_0(M-1) = \text{multiple of } N .$$

In this way, the shifting of the frequency band of interest to baseband is equivalent to rotation of $X_l(k)$ to the left by k_0 samples, without the use of phase factors and with hardly any loss in analysis flexibility.

Example: if $N=2048$ and $M=257$, then the choice of $k_0=\text{multiple of } 8$ gives the desired result.

4. Filter and decimate each segment: that is, form

$$X_{Fl}^\dagger(k) = X_l((k+k_0))_N H^\dagger(k)$$

where

$$H^\dagger(k) = \begin{cases} 0 & \text{in the stopband at } k = N/(2D) , \dots, N - N/(2D) - 1 ; \\ H(k), & \text{the Kaiser frequency response, in the passband at } k=0,\dots, \\ & N/(2D) - 1 \text{ and } k = N - N/(2D), \dots, N-1 \end{cases}$$

where $H(k)$ = N -point FFT of $h(n)$ of length M which is augmented by $N-M$ zeros. In practice, the values of $H^+(k)$ of length N/D (the generally non-zero values) are precomputed and stored for different resolution requirements. The filtered and decimated output is

$$x_{F\ell}(nD) = N/D - \text{point IDFT of } X_{F\ell}^+(k)$$

in which the stopband values of $X_{F\ell}^+(k)$ are discarded. Two additional constraints are required:

- (i) N must be divisible by $2D$ in order to form $X_{F\ell}^+(k)$, and
- (ii) $M-1$ must be divisible by D because, in forming the unbroken filtered and decimated output from $x_{F\ell}(nD)$, the first $(M-1)/D$ points are invalid and discarded, and the last $(N-M+1)/D$ points of $x_{F\ell}(nD)$ for $\ell=0,1,\dots,L-1$ are concatenated to give $x_D(m)$.

5. Finally, form the N -point FFT of $x_D(m)$ and smooth the result in the frequency domain with Hanning weights. Appropriate averaging of such N -point results - from overlapping segments in $x_D(m)$ - should be done to obtain statistically reliable spectral estimates.

6. CONCLUSIONS

An efficient high-resolution spectral analysis technique has been developed by combining the filtering and decimation procedures in the frequency-domain into a single economical operation and by using a suboptimal filter response that preserves the analysis dynamic range. The salient points to be noted are as follows:

- (i) The aliasing error arising from conventional digital filtering and decimation operations is the primary factor limiting the achievable analysis dynamic range.
- (ii) The aliasing arising from the wraparound error produced by the sampled-data filter of length N , which exceeds the length of the overlap in the input data segments, turns out to be a second order effect relative to the primary aliasing error mentioned in (i) above.
- (iii) To enhance the analysis dynamic range, smoothing of the spectral estimates should be carried out, such as by the use of Hanning weights.
- (iv) Computationally, the method compares favourably with the familiar and efficient FFT algorithm operating on an equivalently long data sequence to produce the same frequency resolution. With this method, increased resolution over several smaller finite bandwidths of interest is exchanged for low resolution over a wide bandwidth. More importantly, this high resolution technique can be implemented on state-of-the-art hardware processors having limited FFT lengths. The advantages of this technique make it very attractive for use in real-time hardware processors to meet the demanding spectral analysis tasks encountered in computationally intensive practical applications.



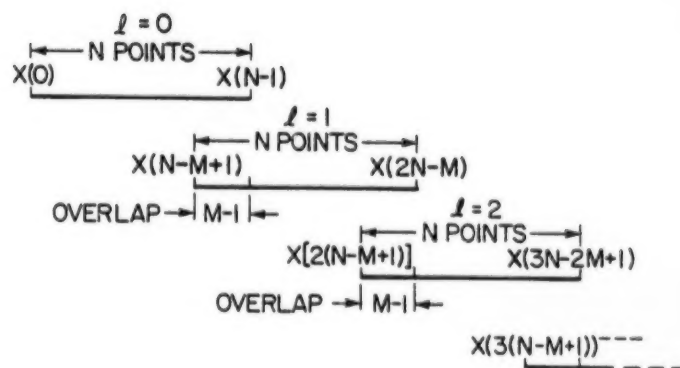
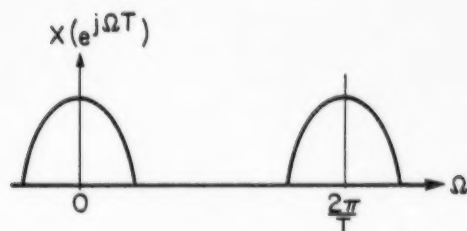
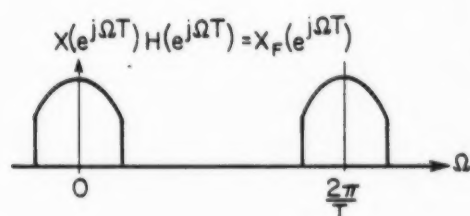


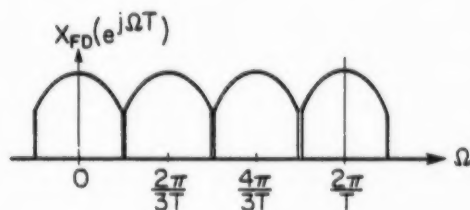
FIG. 1. Segmentation of data $x(n)$ where l = segment number. Each segment contains N points and overlaps the adjacent segment by $M-1$ points.



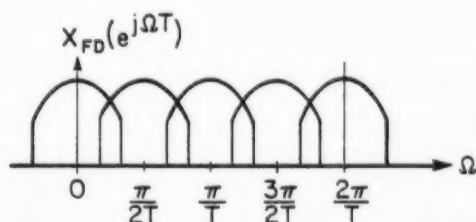
2a. Spectrum of $x(n)$ after heterodyning to baseband.



2b. Spectrum after lowpass filtering.

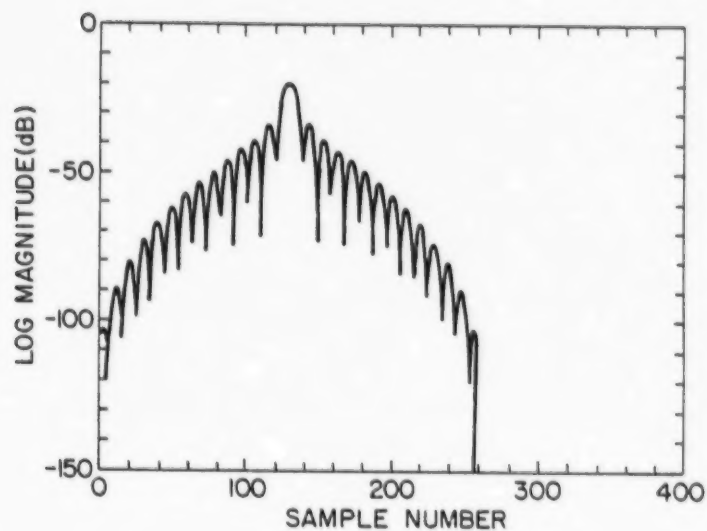


2c. Spectrum after filtering (2b) and decimation by $D=3$.

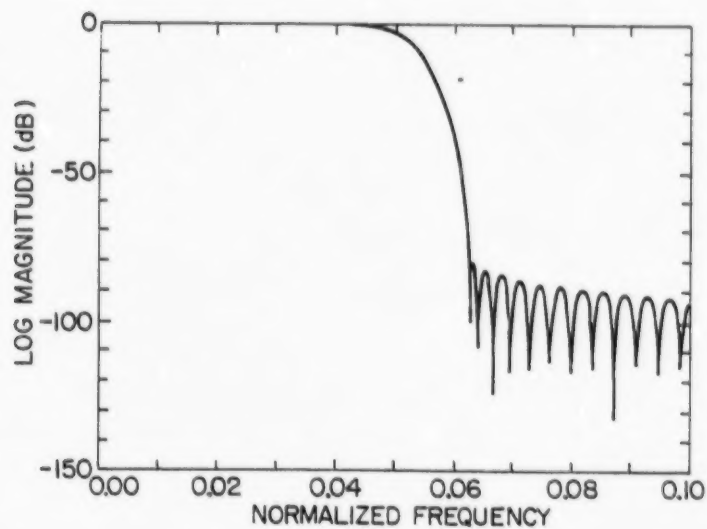


2d. Spectrum after filtering (2b) and decimation by $D=4$.

FIG. 2. Possible aliasing arising from decimation.

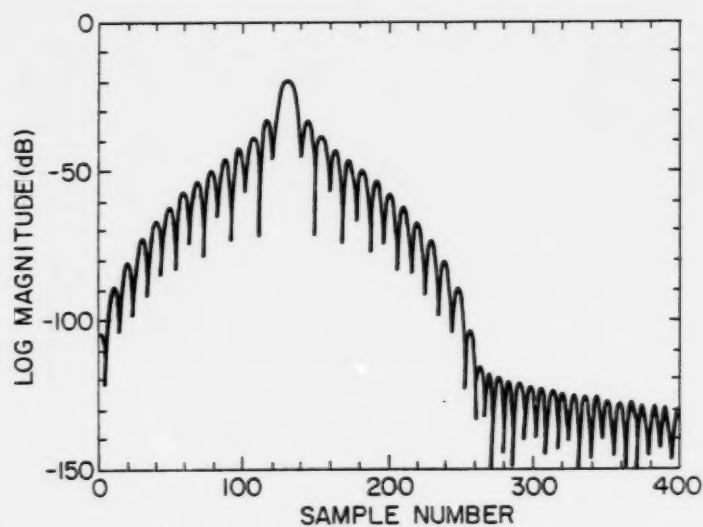


(a) Impulse Response $h(n)$

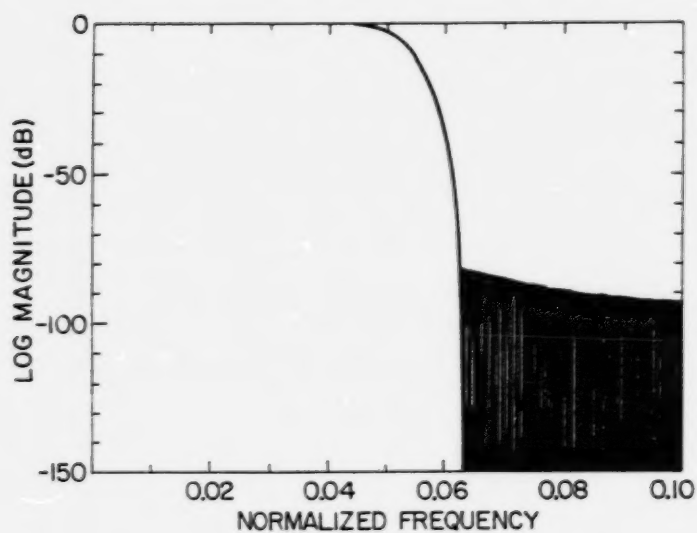


(b) Frequency Response $H(k)$

FIG. 3. Kaiser window lowpass filter for $M=257$ and $\beta=7.865$.



(a) Impulse Response $h^{\dagger}(n)$



(b) Frequency Response $H^{\dagger}(k)$

FIG. 4. Sampled-data FIR filter, having passband frequency response equal to that of Kaiser LP filter and $M=2048=N$.

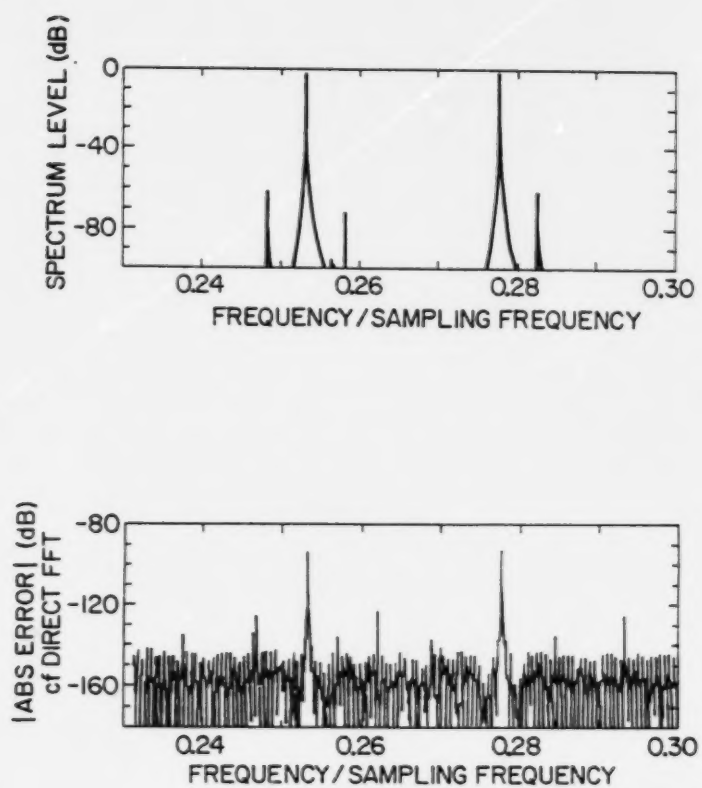


FIG. 5. An analysis of five (5) sinusoids using the filter in Fig. 4.

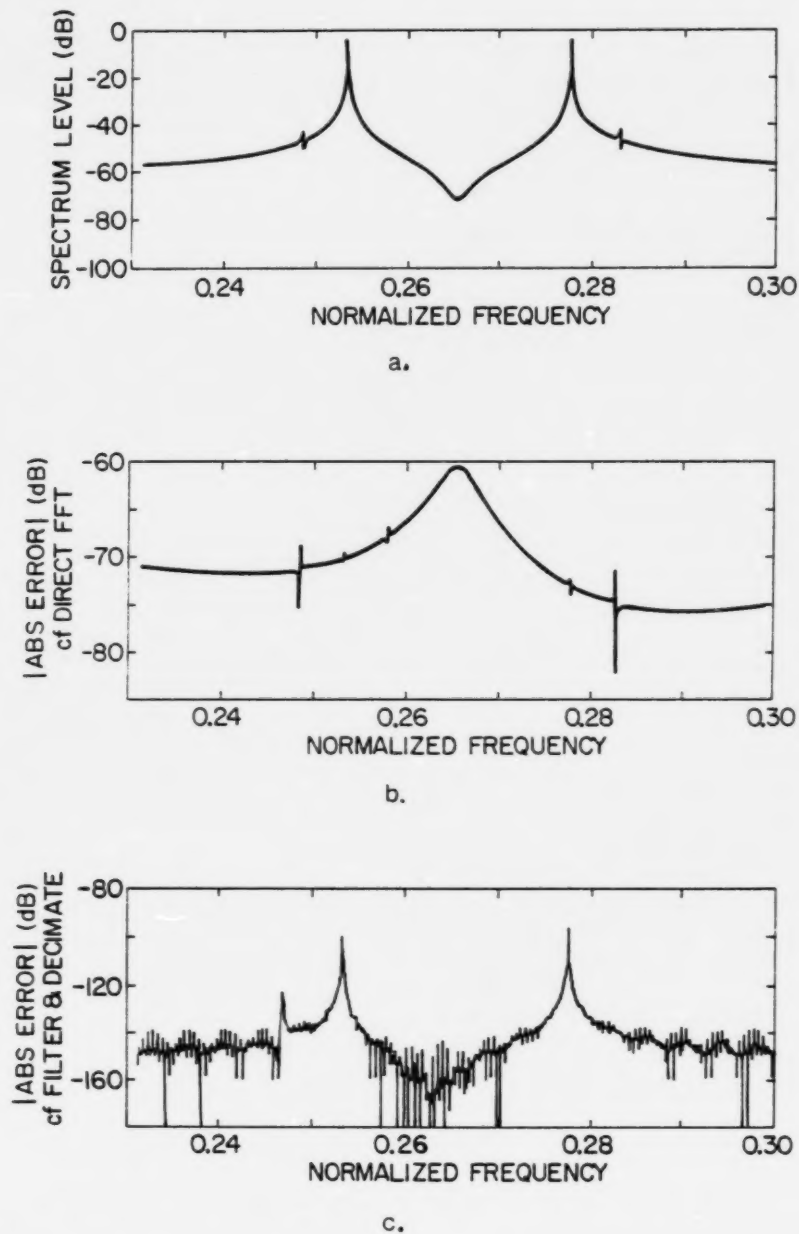


FIG. 6. Unsmoothed results corresponding to the same example as in Fig. 5, using the filter in Fig. 4 on five (5) sinusoidal signals.

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APPENDIX A

KAISER WINDOW LOWPASS FIR DIGITAL FILTER

The ideal lowpass filter has the impulse response coefficients

$$h(n) = \frac{\sin(2\pi n f_c / f_s)}{\pi n}, \quad -\infty < n < \infty, \quad (A-1)$$

where f_c denotes the cutoff frequency and f_s the sampling frequency. This impulse response, however, is infinite in length and unrealizable because the impulse response begins at $-\infty$; that is, the result is an unrealized IIR (Infinite Impulse Response) filter.

To obtain an FIR (Finite Impulse Response) design, (A-1) is truncated at $N=\pm(M-1)/2$. Since the coefficients in (A-1) are in reality Fourier series coefficients, the truncation produces the familiar Gibbs phenomenon: that is, the truncation produces an unacceptably large ripple at the very edge of the passband of the filter frequency response.

A successful FIR filter results from a modification of the coefficients in (A-1) by a finite weighting sequence $w(n)$ - called a window - to control the convergence of the Fourier series. That is, the impulse response coefficients of the FIR filter are given by

$$h(n)w(n) \quad (A-2)$$

where

$$w(n) \equiv 0 \text{ for } +\frac{M-1}{2} < n < \frac{M-1}{2} \quad (A-3)$$

A simple but effective window is the so-called Kaiser window, which is of the form

$$w(n) = \frac{I_0(\beta \sqrt{1 - [2n/(M-1)]^2})}{I_0(\beta)}, \quad \text{for } -\frac{M-1}{2} \leq n \leq \frac{M-1}{2}, \quad (A-4)$$

The function $I_0(x)$ is the modified Bessel function of order zero and is easily computed by means of the power series expansion

$$I_0(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{(x/2)^k}{k!} \right]^2.$$

The quantity β is a constant that specifies a frequency response tradeoff between the peak height of the sidelobe ripples and the

width or energy of the main lobe. To obtain a better approximation to the ideal lowpass filter, the width of the transition band is increased in order to decrease the peak error of the ripple. In the Kaiser window, the parameter β allows a tradeoff between the transition width α (in units of M) and the peak ripple. Typical values of β , α , passband ripple and stopband ripple are:

β	α	Passband Ripple (dB)	Stopband Ripple (dB)
2.120	1.50	± 0.27	-30
3.384	2.23	± 0.0864	-40
4.538	2.93	± 0.0274	-50
5.658	3.62	± 0.00868	-60
6.764	4.32	± 0.00275	-70
7.865	5.0	± 0.000868	-80
8.960	5.7	± 0.000275	-90
10.056	6.4	± 0.000087	-100

(See Reference 23, Sections 3.11 and 3.12).

Essentially, the Kaiser window is optimum in the sense that it is a finite duration sequence having the minimum spectral energy beyond some specified frequency. As a result, the FIR lowpass digital filter using the Kaiser window design can be expected to exhibit suboptimal properties in filtering and decimation operations.

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13. ABSTRACT A modified version of the complex demodulation approach is used in the development of an efficient high-resolution technique for spectral analysis. Through a combined filtering and decimation operation and a suboptimal Finite Impulse Response (FIR) digital filter, the Fast Fourier Transform (FFT) sizes in the fast convolutions are reduced significantly. Though this procedure introduces wraparound effects, such errors are small compared to aliasing arising from the normal digital filtering and decimation operations. Aside from its capability to analyze multiple finite bandwidths with different frequency resolutions, the accuracy obtainable by this method allows for a large analysis dynamic range.		

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